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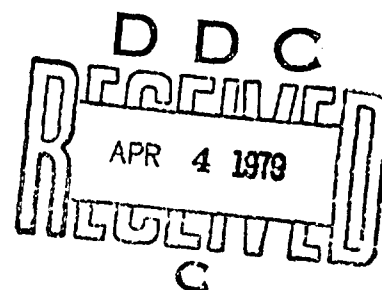
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Approximate Tests of Hypotheses in Regression  
Models with Grouped Data

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## 1. Introduction

In many real situations, observations are available only in certain groups. This process of recording or storing observations in groups directly leads to grouped data. Experimental observations where precise measuring instruments are not available also result in grouped data. Various authors have provided examples of diverse fields where statistical analysis has to depend on grouped data, for reference, see Indrayan and Rustagi (1979). In that paper, the case of approximate maximum likelihood estimates was discussed for regression models. In this paper, we provide techniques for testing hypotheses about parameters in the regression model under the situation of grouped data. We consider a test statistic which is similar to the conventional F statistic for the ungrouped case. A simulation study, performed for a few cases, shows that the proposed statistic has an approximate F-distribution.

The approximation here is to the order  $O(h^4/\sigma^4)$  where  $h$  is the length of the interval of recorded observations and  $\sigma$  is the standard deviation. In a way, this simulation study confirms the robustness of the F-statistic for the regression models in the grouped case and is likely to be very useful in applications.

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## 2. Model and Notation

The notation used here is the same as in Indrayan and Rustagi (1979). It is assumed that there are K populations, generated by random variables  $y_1, y_2, \dots, y_K$ . Further let  $y_k$  be normally distributed with mean  $\mu_k$  and variance  $\sigma^2$ , where

$$\begin{aligned}\mu_k &= \beta_1 x_{k1} + \beta_2 x_{k2} + \dots + \beta_p x_{kp}, \\ k &= 1, 2, \dots, K,\end{aligned}\tag{2.1}$$

$\beta_i$ 's are the unknown parameters and x's are known constants. It is assumed that there are  $n_k$  independent observations in  $y_k$ , denoted as  $y_{uk}$ ,  $u = 1, 2, \dots, n_k$  with  $\sum_{k=1}^K n_k = n$ . The matrix of observations denoted by

$$\begin{aligned}\underline{X} &= (x_{ij}) \\ i &= 1, 2, \dots, n, \\ j &= 1, 2, \dots, p,\end{aligned}$$

is known.

Suppose the possible values of the random variables  $y_i$  are recorded in intervals  $[C_{i-1}, C_i)$ ,  $i = \dots, -2, -1, 0, 1, 2, \dots$  with  $C_i - C_{i-1} = h$ . Let  $N_{ik}$  be the number of observations on  $y_k$  in the interval  $[C_{i-1}, C_i)$  and let this probability be  $\pi_{ik}$ .

Let  $\underline{C}$  be a matrix of order  $m \times p$  of known constants and let  $\underline{\beta}' = (\beta_1, \dots, \beta_p)$ . We are interested in the test of the hypothesis

$$H_0: \underline{C}\underline{\beta} = \underline{a}\tag{2.2}$$

versus the alternative

$$H_1: \mathbf{C}\mathbf{B} \neq \mathbf{a}$$

for some given constant vector  $\mathbf{a}$ .

In the usual ungrouped case, where  $K$  populations are tested for the above hypothesis, the analysis of variance test utilizing the  $F$ -distribution is generally available in most books, see for example, Rao (1973).

For the grouped data case, the likelihood of the sample  $L(\boldsymbol{\pi})$  is obtained in terms of multinomials.

Let the maximum likelihood estimates of  $\pi_{ik}$  under the null hypothesis be denoted by  $\pi_{ik}(\hat{\omega})$  and under the full model by  $\pi_{ik}(\hat{\Omega})$ . Maximum likelihood estimates were discussed in an earlier paper by the authors (1979). The likelihood ratio given by

$$\lambda = \frac{L(\pi_{ik}(\hat{\omega}))}{L(\pi_{ik}(\hat{\Omega}))} \quad (2.3)$$

is used to provide tests for the hypotheses (2.2). The asymptotic distribution of  $-2 \log \lambda$  follows chi-squared distribution as in the ungrouped case under the approximation ignoring terms of  $O(\frac{h_4}{\sigma})$ .

### 3. F-statistic

In the usual ungrouped case, the test of the general linear hypothesis is obtained in terms of an F test. For the sake of completeness, we state below the Fundamental Lemma of Analysis of Variance, Rao (1973).

Lemma 3.1. For the Gauss-Markov model  $(Y, X\beta, \sigma^2 I)$ , the test of hypothesis  $C\beta = a$  where  $C$  is a given  $m \times p$  matrix of rank  $m$ , is given by the statistic

$$F = \frac{R_1^2 - R_0^2}{m} \div \frac{R_0^2}{n - r} \quad (3.1)$$

where  $r = \text{rank of matrix } X$  with

$$R_0^2 = \min_{\beta} (Y - X\beta)'(Y - X\beta),$$

and

$$R_1^2 = \min_{C\beta = a} (Y - X\beta)'(Y - X\beta).$$

The statistic (3.1) has an F-distribution with  $p$  and  $n-r$  degrees of freedom.

The test of hypothesis in the grouped-data case can be similarly obtained in terms of a statistic which is the ratio of sums of squares. Let the mid point variable  $M$  be defined by the following:

$$M = m_i \text{ if and only if } Y \in [C_{i-1}, C_i) \\ i = \dots -2, -1, 0, 1, 2, \dots$$

The approximate maximum likelihood estimate of  $\beta$  is given by

$$\hat{\beta}_0 = (X'X)^{-1} X'M \quad (3.2)$$

where  $\underline{A}^-$  denotes the generalized inverse (g-inverse) of the matrix  $\underline{A}$ , for reference, see Rao and Mitra (1971) and  $\underline{M}$  is the vector of mid points resulting from the data. The mid point estimator  $\hat{\underline{\beta}}_0$  leads to a statistic

$$F_0 = \frac{R_{LM}^2 - R_{OM}^2}{m} \div \frac{R_{OM}^2}{n - r} \quad (3.3)$$

where  $R_{OM}^2 = \min_{\underline{\beta}} (\underline{M} - \underline{X}\underline{\beta})'(\underline{M} - \underline{X}\underline{\beta})$ , and

$$R_{LM}^2 = \min_{\underline{C}\underline{\beta} = \underline{a}} (\underline{M} - \underline{X}\underline{\beta})'(\underline{M} - \underline{X}\underline{\beta}).$$

As usual, we evaluate the sums of squares in the statistic (3.3) by using the following notation.

$$\text{Suppose } \begin{pmatrix} \underline{I} & \underline{X} \\ \underline{X}' & \underline{Q} \end{pmatrix} = \begin{pmatrix} \underline{T}_1 & \underline{T}_2 \\ \underline{T}_3 & -\underline{T}_4 \end{pmatrix},$$

then

$$R_{OM}^2 = \underline{M}'\underline{T}_1\underline{M} \quad (3.4)$$

$$R_{LM}^2 - R_{OM}^2 = \frac{(\underline{C}'\underline{M} - \underline{a})'(\underline{C}'\underline{C})^{-1}(\underline{C}'\underline{M} - \underline{a})}{(\underline{C}'\underline{C})} \quad (3.5)$$

To ensure estimability of  $\underline{C}\underline{\beta}$  we assume that  $\underline{C} = \underline{A}'\underline{X}$  for some  $\underline{A}$ , with rank of  $\underline{C} = m$ .

Without loss of generality, the statistic  $F_0$  can be considered for the case of testing the hypothesis that  $\underline{\beta} = \underline{0}$ . In that case we have

$$\begin{aligned} R_{OM}^2 &= \min_{\underline{\beta}} (\underline{M} - \underline{X}\underline{\beta})'(\underline{M} - \underline{X}\underline{\beta}) \\ &= (\underline{M} - \underline{X}\hat{\underline{\beta}}_0)'(\underline{M} - \underline{X}\hat{\underline{\beta}}_0) \\ &= \underline{M}'\underline{M} - \hat{\underline{\beta}}_0'\underline{X}'\underline{M}, \end{aligned}$$



and  $R_{1M}^2 = \tilde{M}'\tilde{M}$  so that  $R_{1M}^2 - R_{0M}^2 = \tilde{B}_0' \tilde{X}'\tilde{M}$ . Usually  $R_{0M}^2$  is called the Error Sum of Squares (Error SS) and  $R_{1M}^2 - R_{0M}^2$  as the Regression Sums of Squares (Regression SS).

To show that the asymptotic distribution of

$$F_0 = \frac{(\text{Regression SS})^2}{m} \div \frac{(\text{Error SS})^2}{n-m} \quad (3.6)$$

is chi-squared to the order of approximation implied by Sheppard's corrections, Cramér (1974, pp. 359-362) which is assumed to be negligible here, we have the following lemmas.

Lemma 3.2. The asymptotic distribution of

$$\hat{\beta}_0 = (\tilde{X}'\tilde{X})^{-1} \tilde{X}'\tilde{M} \quad (3.7)$$

as  $n_k \rightarrow \infty$  is p-variate normal with mean  $\beta_0$  and variance covariance

$(\tilde{X}'\tilde{X})^{-1} (\sigma^2 + \frac{h^2}{12})$  to the order to the order of approximation implied by Sheppard's correction.

Proof: Let  $\tilde{M} = \begin{pmatrix} M_1 \\ M_2 \\ \vdots \\ M_K \end{pmatrix}$

where  $M_k$  is  $n_k$ -vector,  $k = 1, 2, \dots, K$ . Suppose  $(\tilde{X}'\tilde{X})^{-1} \tilde{X}' = \tilde{B} = (\tilde{B}_1, \dots, \tilde{B}_K)$  with  $\tilde{B}_1$  being a  $p \times n_1$  matrix,  $1 = 1, 2, \dots, K$ . With the above definitions,

$$\beta_0 = \sum_{i=1}^K \tilde{B}_i M_i \quad (3.8)$$

Let the elements of the partitions  $\tilde{B}_i$  of the matrix  $\tilde{B}$  be denoted by

$\tilde{B}_{lm(1)}$ ,  $l = 1, 2, \dots, p$ ,  $m = 1, 2, \dots, n_1$ . Notice that the first  $n_1$  columns of  $\tilde{X}'$  are identical, the next  $n_2$  columns are also identical and so on.

Therefore, the columns of the matrix  $\tilde{B}_i$  are all identical for  $i = 1, 2, \dots, K$ .

$B_1 M_1$  for each  $i$  reduces to a constant multiple of the sum of  $n_1$  elements of the vector  $M_1$ . Since the components of this sum are independent and identically distributed, by the application of Central Limit Theorem, the vector  $B_1 M_1$  is asymptotically  $p$ -variate normal for each  $i$ . The distribution of  $\hat{\beta}_0$  is consequently also  $p$ -variate normal.

Since  $E(M_1) = E(Y_1)$  and variance of the components of  $M_1$  is  $\sigma^2 + \frac{h^2}{12}$  (subject to the approximation implied by Sheppard's correction), it follows that

$$E(\hat{\beta}_0) = \beta \quad (3.9)$$

and

$$\text{Cov}(\hat{\beta}_0) = (X'X)^{-1} (\sigma^2 + \frac{h^2}{12}) . \quad (3.10)$$

The proof of lemma is now complete.

Lemma 3.3. To the order of approximation implied by the Sheppard correction, the mean and variance of error sums of squares are given by

$$E(\text{Error SS}) = (n-p)\sigma_0^2, \quad (3.11)$$

$$V(\text{Error SS}) = 2(n-m)\sigma_0^4 + \left(\frac{h^4}{80\sigma^4} + \frac{h^2}{2\sigma^2}\right)\sigma_0^4 \sum_1 b_{11}^2, \quad (3.12)$$

where

$$\sigma_0^2 = \sigma^2 + \frac{h^2}{12}, \quad (3.13)$$

and

$$B = (b_{1j}) = I - X(X'X)^{-1} X'. \quad (3.14)$$

Proof. Applying results of theorem 1 of Searle (1971, p. 55) and (3.10'), we have

$$E(\text{Error SS}) = n\sigma_0^2 + \cancel{B'X'XB} - (n\sigma_0^2 + \cancel{B'X'XB}) = (n-m)\sigma_0^2$$

In a paper by Hsu (1938), it has been shown that a quadratic form  $Q = \sum_{ij} a_{ij} Z_i Z_j$ , with  $E(Q) = 1$  and  $\text{Var}(Z_i) = \sigma^2$  for all  $i$ , has the following properties:

$$\text{Var } Q = \sum_i (\mu_{4i} - 3) a_{ii}^2 + 2\sigma^4 \sum_{ij} \sum_{ij} a_{ij}^2$$

where  $\mu_{4i} = E(Z_i - E(Z_i))^4$ .

Suppose now,

$$Q = \frac{\text{Error SS}}{(n-p)\sigma_0^2}$$

so that  $E(Q) = 1$ . Also the matrix  $B$  is symmetric and idempotent with rank  $n-m$  and hence  $\sum_{ij} b_{ij}^2 = n-m$ . By assumptions of normality and using Sheppard's corrections, we have

$$E(M_i - E(M_i))^4 = 3\sigma^4 + \frac{h^2}{2}\sigma^2 + \frac{h^4}{80}$$

for all  $i$ . The result (3.12) follows.

From lemma 3.2, we know that the distribution of  $(\text{Regression SS})/\sigma_0^2$  as  $n_k \rightarrow \infty$ , has a non-central chi-squared distribution with  $p$  degrees of freedom with noncentrality parameter

$$\delta^2 = \frac{1}{2} (\cancel{B'X'XB}) \quad (3.15)$$

to the order of approximation implied by Sheppard's corrections. Further

since  $\sum_i b_{ii}^2 \leq n-m$ , it follows from (3.11) and (3.12) that  $(\text{Error SS})/(n-m)\sigma_0^2$  tends to 1 in probability. Hence  $mF_0$  as given in (3.3) has a noncentral chi-squared distribution with noncentrality parameter  $\delta$ .

Note that under the null hypothesis,  $\delta = 0$ , hence the asymptotic distribution of  $F_0$  is central chi-squared. Therefore the test can be easily performed.

For small  $h$ , the distribution of  $F_0$  may turn out to be close to the F-distribution. Box and Anderson (1955) have developed robust tests for non-normal populations using the following. Assume that the distribution of  $(\text{Error SS})/(n-m)\sigma_0^2$  is  $\chi_v^2/v$  where degrees of freedom  $v$  are obtained by a method of moments given by

$$v = n-m - \frac{h^2}{4\sigma^2} \sum_i b_{ii}^2 + O\left(\frac{h^4}{\sigma^4}\right) \quad (3.16)$$

For small  $h/\sigma$ , we have hardly any correction to degrees of freedom and then the test can be performed as an F test. This behavior of the statistic  $F_0$  has been studied through simulations and goodness of the approximation is measured in terms of Kolmogoroff-Smirnov statistic in the next section.

#### 4. Simulations

Two models have been considered for simulations.

Model I.

$$Y_{uk} \sim N(\alpha + \beta X_k, \sigma^2)$$

$$u = 1, 2, \dots, n_k; k = 1, 2, \dots, K.$$

We consider  $K = 10$ , with  $X_1 = 0, X_2 = 1, \dots, X_{10} = 10, n_k = 100$  (same sample sizes for every  $k$ ). We consider two cases of  $\alpha, \beta$ ,

$$\alpha = 10, \quad \beta = 0$$

$$\alpha = 40 \text{ and } \beta = 0.$$

The values of  $\sigma^2$  are chosen to be

$$25 \text{ or } 100.$$

The size of  $h$  is taken as

$$0, 2, 3, 4, 5, 10, 15, 20.$$

The hypothesis considered here is

$$H_0: \beta = 0$$

Using the usual IBM Random Number Generator package, samples were generated and then were grouped in intervals of size  $h$ . The  $F$  statistic was calculated for the ungrouped data case and  $F_0$  statistic for the various grouped data cases. The empirical cumulative distribution of the statistics were then computed and were compared with the theoretical  $F$ -distribution using Kolmogorov-Smirnov distribution. Table I and II describe the results of the simulations for

$n_K = 10$  and  $n_K = 100$  respectively. The last column gives the tail probability for significance in both cases. The column with heading D gives the actual value of Kolmogorov-Smirnov statistic.

Table I

h	$v_1$	$v_2$	D	P
----- $\sigma^2=25, \alpha=10$ -----				
0	1	98	0.6120	0.8481
2	1	98	0.6311	0.8206
3	1	98	0.6988	0.8658
4	1	98	0.4953	0.9669
5	1	98	0.8531	0.4606
10	1	98	1.1812	0.1227
15	1	98	0.8534	0.4602
20	1	98	1.1241	0.1597
----- $\sigma^2=25, \alpha=40$ -----				
0	1	98	0.9169	0.3699
2	1	98	0.9339	0.3477
3	1	98	0.8668	0.4401
4	1	98	0.6613	0.7743
5	1	98	0.7018	0.7082
10	1	98	0.9671	0.3070
15	1	98	0.8695	0.4362
20	1	98	1.2566	0.0850
----- $\sigma^2=100, \alpha=10$ -----				
0	1	98	0.6721	0.7570
2	1	98	0.8685	0.4376
3	1	98	0.7632	0.6050
4	1	98	0.5060	0.9600
5	1	98	0.5713	0.8999
10	1	98	0.6226	0.8330
15	1	98	0.7894	0.5614
20	1	98	0.8204	0.5114
----- $\sigma^2=100, \alpha=40$ -----				
0	1	98	0.7245	0.6701
2	1	98	0.7987	0.5463
3	1	98	0.8099	0.5281
4	1	98	0.8101	0.5278
5	1	98	0.7078	0.6982
10	1	98	0.8516	0.4629
15	1	98	0.9647	0.3098
20	1	98	0.6869	0.7330

Table II

h	$v_1$	$v_2$	D	P
----- $\sigma^2=25, \alpha=10$ -----				
0	1	998	0.8214	0.5097
2	1	998	0.9069	0.3832
3	1	998	0.8081	0.5310
4	1	998	0.7360	0.6507
5	1	998	0.8411	0.4790
10	1	998	0.4508	0.9871
15	1	998	0.9026	0.3891
20	1	998	0.7345	0.6532
----- $\sigma^2=25, \alpha=40$ -----				
0	1	998	0.5086	0.9562
2	1	998	0.7267	0.6664
3	1	998	0.6666	0.7659
4	1	998	0.5544	0.9184
5	1	998	0.6424	0.8037
10	1	998	0.5871	0.8809
15	1	998	0.7458	0.6342
20	1	998	1.1031	0.1753
----- $\sigma^2=100, \alpha=10$ -----				
0	1	998	0.7605	0.6096
2	1	998	0.7578	0.6140
3	1	998	0.7627	0.6058
4	1	998	0.9348	0.3466
5	1	998	0.8685	0.4376
10	1	998	0.6699	0.7606
15	1	998	0.6582	0.7792
20	1	998	0.9366	0.3442
----- $\sigma^2=100, \alpha=40$ -----				
0	1	998	0.7652	0.6017
2	1	998	0.7107	0.6934
3	1	998	0.7613	0.6082
4	1	998	0.7483	0.6300
5	1	998	0.5413	0.9313
10	1	998	0.5698	0.9015
15	1	998	0.8149	0.5201
20	1	999	0.5546	0.9182



Model II. We consider here the regression model

$$Y_{uk} \sim N(\beta_1 X_{k1} + \beta_2 X_{k2} + \beta_3 X_{k3}, \sigma^2)$$
$$k = 1, 2, \dots, 4.$$

Four sets of  $X_{ku}$  are utilized

(i)	$X_{11} = 0$	$X_{12} = 1$	$X_{13} = 3$
(ii)	$X_{21} = 2$	$X_{22} = 4$	$X_{23} = 6$
(iii)	$X_{31} = 7$	$X_{32} = 5$	$X_{33} = 3$
(iv)	$X_{41} = 9$	$X_{42} = 1$	$X_{43} = 8$

$$h = 0, 10, 50, 100, 200$$

$$\sigma = 100$$

$$n_k = 10, 25, 100 \quad (\text{same for all } k)$$

$$\beta_1 = \beta_2 = \beta_3 = 0.$$

The hypothesis tested here is

$$H_0: \beta_2 = \beta_3 = 0$$

The results are given in Table III and are based on 100 samples.

Table III

h	$v_1$	$v_2$	D	P
----- $n_k = 10$ for all k -----				
0	2	37	0.7567	0.6158
10	2	37	0.6984	0.7138
50	1	37	0.7600	0.6104
100	2	37	0.7239	0.6712
200	2	37	1.2845	0.0738
----- $n_k = 25$ for all k -----				
0	2	97	0.8039	0.5378
10	2	97	0.8742	0.4294
50	2	97	0.8401	0.4805
100	2	97	0.7487	0.6293
200	2	97	0.9388	0.3415
----- $n_k = 100$ for all k -----				
0	2	397	0.7705	0.5927
10	2	397	0.7468	0.6325
50	2	397	0.8749	0.4284
100	2	397	1.1426	0.1469
200	2	397	1.1312	0.1546

## 5. Discussion

Comparison of the tail probability of the Kolmogorov-Smirnov statistic for  $h = 0$  (that is, ungrouped case) with various values of  $h$ , shows that on the whole, the empirical cumulative distributions are the same for the cases  $h/\sigma \leq 1$ . When  $\frac{h}{\sigma} > 1$ , we do not have very good results. In general, one could make the statement that the F-distribution is fairly a good approximation to the distribution of the statistic  $F_0$ . More extensive simulations may be able to provide further evidence of this correspondence.

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